Generation, compression, and propagation of pulse trains in the nonlinear Schrödinger equation with distributed coefficients

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The generalized nonlinear Schrödinger model with distributed dispersion, nonlinearity, and gain or loss is considered and the explicit, analytical solutions describing the dynamics of bright solitons on a continuous-wave background are obtained in quadratures. Then, the generation, compression, and propagation of pulse trains are discussed in detail. The numerical results show that solitons can be compressed by choosing the appropriate control fiber system, and pulse trains generated by modulation instability can propagate undistorsted along fibers with distributed parameters by controlling appropriately the energy of each pulse in the pulse train.

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Solitons constitute a new paradigm in nonlinear science; they are ubiquitous, fundamental objects of nature. Solitons, or more properly solitary waves, manifest themselves in a large variety of systems in nature, namely in any physical system that possesses both dispersion (or diffraction) and nonlinearity. It is believed that they will play a major role in the modern nonlinear science, being identified in diverse fields such as optics, plasmas, fluid mechanics, condensed matter, matter waves, particle physics, and astrophysics. However, over the past two decades, the forefront of soliton research has shifted to optics and quite recently to the field of matter waves (Bose-Einstein condensates); for some recent overviews of optical solitons (spatial, temporal, and spatiotemporal solitons) see Refs. [1–3].

Optical solitons are regarded as the natural carriers of information (that is, the natural data bits) and they constitute an important alternative for the next generation of ultrahigh-speed optical telecommunication systems. The propagation of optical solitons in optical fibers is adequately described by the nonlinear Schrödinger (NLS) equation, which is a universal model in the whole nonlinear science. The fiber solitons that are the prototype of temporal optical solitons have been the object of extensive theoretical and experimental studies during the last three decades [4–9].

In the past years, a lot of attention has been focused to the study of generalized higher-order NLS equations containing terms accounting for specific physical effects such as third-order dispersion, self-steepening, and delayed nonlinear response, and new exact solutions have been found, such as N-soliton solutions of the bright type [10–17], N-soliton solutions of the dark type [18–21], the combined solitary wave solutions [22,23], and soliton solutions on a continuous-wave (cw) background [24–26]. It should be noted that all of these solutions mentioned above were obtained for the ideal optical fiber transmission system, that is, for a generalized NLS equation with constant coefficients. However, in a real fiber, in general, the core medium is not homogeneous. There are always present some fiber nonuniformities to influence various effects such as gain or loss, group-velocity dispersion (GVD), and self-phase modulation, etc. The prototype equation for describing such effects is therefore the generalized NLS equation with *distributed coefficients*, accounting for varying dispersion, nonlinearity, and gain or absorption. Novel exactly integrable NLS equations with distributed coefficients have been discovered [27–29] as useful models to design novel dispersion-managed fiber transmission systems [30–35]. It is believed that transmission of return-to-zero optical pulses in dispersion-managed fibers is a key technology in ultrahigh bit rate optical communication systems. The strong dispersion management technique offers the possibility to control fiber nonlinearity and to suppress specific nonlinear effects such as the self-phase modulation and interchannel cross-talk in wavelength-division-multiplexed fiber systems. Suppression of the latter undesirable effect is a key issue to overcome in order to achieve bit rates of 40 Gb/s, or even more. We consider the soliton management problem as described by the NLS equation with varying coefficients,

$$i\frac{\partial E}{\partial z}+\frac{D(z)}{2}\frac{\partial^2 E}{\partial t^2}+R(z)|E|^2E+i\Gamma(z)E=0, \tag{1}$$

where $E(z,t)$ is the complex envelope of the electrical field in the moving frame, z is the normalized distance of propagation along the fiber, t is the retarded time, $D(z)$ represents the GVD coefficient, $R(z)$ is the nonlinearity parameter, and $\Gamma(z)$ is the amplification or absorption coefficient. Thus, the above NLS equation governs the propagation dynamics of optical pulses in a fiber with distributed dispersion, nonlinearity, and gain or loss. It describes, for example, the amplification or absorption of pulses propagating in a single mode optical fiber with distributed GVD and self-focusing Kerr

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nonlinearity. In practical applications, this model is of primary interest, not only for the study of amplification/absorption and compression/broadening of optical solitons in inhomogeneous systems, but also for the study of dispersion-managed transmission systems. Equation (1) with various forms of inhomogeneities has been extensively studied in recent works [27–29,36–40]. The nonlinear compression of chirped solitary waves and quasisoliton propagation in dispersion management optical fibers with the phase modulation or the gain/loss terms have been discussed in detail [36–39]. Exact solutions [27,28,40] and the so-called self-similar soliton solutions [29] were obtained as expressed in quadratures. However, less investigated was the problem of generation, compression, and propagation of soliton trains in such systems described by the NLS equation with distributed coefficients.

Our aim in this paper is to present, in a closed and analytic form, the bright soliton solution on a continuous-wave background corresponding to the NLSE equation (1) with varying coefficients and to study in detail the generation, compression, and propagation of soliton pulse trains. Thus we consider the NLS equation (1) with distributed coefficients, and by using direct transformations of variables and functions we arrive at the universal NLS with constant coefficients for which the exact soliton solution on a cw background is well known; see, e.g., Refs. [8,41–43]. Then by making the reverse transformation of variable and functions we get the most general expression of the soliton on a cw background for the NLS equation (1) with distributed GVD, nonlinearity, and gain or loss. The extensive numerical results show that the pulse train can be compressed by choosing the appropriate control fiber system, and the pulse train generated by the modulation instability effect can propagate undistorded (the intrachannel cross-talk being suppressed) along the fiber by choosing the suitable control parameters.

Next we introduce the following transformation of variables and functions:

$$E(z,t) = \Delta(z)\sqrt{\frac{D(z)}{R(z)}}u(\xi,\tau)\exp[iC_0\Delta(z)t^2/2],$$

$$\tau = \Delta(z)t, \quad \xi = \Delta(z)\int_0^z D(\zeta)d\zeta, \tag{2}$$

where $\Delta(z)$ is given by

$$\Delta(z) = \frac{1}{1 + C_0\int_0^z D(\zeta)d\zeta}, \tag{3}$$

and presents the compression/broadening factor [$\Delta(z) > 1$ for compression, and $\Delta(z) < 1$ for broadening]. Here C_0 is an arbitrary real constant, which represents the initial chirp parameter. We impose the following condition to be fulfilled by the coefficients $D(z)$, $R(z)$, and $\Gamma(z)$:

$$\Gamma(z) = \frac{C_0 D(z)\Delta(z)}{2} - \frac{W(R,D)}{2D(z)R(z)}. \tag{4}$$

In the above equation, the function $W(R,D)$, defined as

$$W(R,D) = \dot{D}(z)R(z) - D(z)\dot{R}(z), \tag{5}$$

is the Wronskian of the two functions $R(z)$ and $D(z)$ (for a discussion of this issue see Ref. [27]). Here "." represents the derivative with respect to z.

At this point we notice that the transform (2) does not work for the strong dispersion management regime; it applies only to the moderate dispersion management regime and to distributed gain/loss and fiber nonlinearity [27–29,40]. This is quite evident because we implicitly assumed that $D(z)/R(z) > 0$ and $\Delta(z) > 0$ for all z. For the case of strong dispersion management, typically, the GVD coefficient $D(z)$ changes the sign along the fiber and makes the transform (2) useless. A transformation similar to (2) was known earlier for the case of a constant nonlinear coefficient, $R(z)=\text{const}$ (see, for example, Ref. [6]). However, by controlling the nonlinear refractive index profile along the fiber, it makes possible the variation of the nonlinear coefficient R with z, and then the most general transform (2) written above applies to the case of an optical fiber with tailor-made dispersion and nonlinearity profiles.

Thus, by using the above transformations of functions and variables, Eq. (1) becomes the standard, universal NLS equation $iu_\xi + u_{\tau\tau}/2 + |u|^2u = 0$ for which the bright soliton solution on a cw background is well known; see, e.g., Ref. [8]. Coming back to the original equation with distributed coefficients, Eq. (1), its most general solution representing the bright soliton on a cw background can be written as follows [24,25]

$$E(z,t) = \Delta(z)\sqrt{\frac{D(z)}{R(z)}}Q(z,t)\exp(i\varphi_c), \tag{6}$$

where $Q(z,t)$ is given by

$$Q(z,t) = A + A_s\frac{a\cosh\theta + \cos\varphi + i(b\sinh\theta + c\sin\varphi)}{\cosh\theta + a\cos\varphi},$$

and

$$\theta = M_I\Delta(z)t - \frac{1}{2}[A_sM_R + (\omega+\omega_s)M_I]\Delta(z)\int_0^z D(\zeta)d\zeta - \theta_0,$$

$$\varphi = M_R\Delta(z)t - \frac{1}{2}[(\omega+\omega_s)M_R - A_sM_I]\Delta(z)\int_0^z D(\zeta)d\zeta - \varphi_0,$$

$$\varphi_c = \Delta(z)\left[C_0t^2/2 + \omega t + (A^2 - \omega^2/2)\int_0^z D(\zeta)d\zeta\right].$$

Here $a=-2AA_s/D$, $b=-2AM_R/D$, $c=M_I/A_s$, with $D=A_s^2+M_R^2$, and $M_R+iM_I=[(\omega-\omega_s-iA_s)^2+4A^2]^{1/2}$, which implies that $M_I=0$ when the soliton amplitude $A_s=0$. The parameters θ_0, φ_0, A_s, ω_s, A (the background amplitude), and ω are arbitrary real constants. From Eq. (6) one can see that, as the background amplitude A vanishes, the general solution (6) reduces to the bright single-soliton solution,

$$E(z,t) = A_s\Delta(z)\sqrt{\frac{D(z)}{R(z)}}\,\mathrm{sech}\,\theta_s \exp(i\varphi_s), \tag{7}$$

where $\theta_s = A_s\Delta(z)[t-\omega_s\int_0^z D(\zeta)d\zeta] - \theta_0$, $\varphi_s = \Delta(z)\left[\frac{1}{2}C_0t^2+\omega_s t + \frac{1}{2}(A_s^2-\omega_s^2)\int_0^z D(\zeta)d\zeta\right] - \varphi_0$ and θ_0, φ_0 are the arbitrary real numbers. Here C_0 is an arbitrary real constant, which represents the initial chirp parameter. The soliton amplitude is given by $|E(z,t)| = |A_s|\Delta(z)\sqrt{D(z)/R(z)}\mathrm{sech}\,\theta_s$, and the maximum (peak) amplitude is $A_s\Delta(z)\sqrt{D(z)/R(z)}$, and therefore depends on z.

In the particular case when $D(z)=R(z)=1$, the above general formula reduces to that given in Ref. [39], where some aspects related to soliton compression and interaction between neighboring solitons were investigated in the ideal case of constant GVD and nonlinearity coefficients, however, allowing for a distributed gain or loss. In another particular case when the initial chirp parameter $C_0=0$, the compression/broadening factor $\Delta(z)=1$, the solution (7) reduces to that given previously in Ref. [40]. When $\omega_s=0$, the general solution (7) is similar with the exact sech-type self-similar solution given in Ref. [29]. However, here we present a more general situation. Indeed, from the expression of θ_s one can see that the soliton's width varies due to the presence of the compression/broadening factor $\Delta(z)$.

In order to further understand the unique behavior of the exact solution (7), we take as an example, an exponentially distributed control system. For definiteness we assume that the GVD and the nonlinearity parameter are distributed according to

$$D(z) = d\exp(-gz), \quad R(z) = r\exp(-\sigma z), \tag{8}$$

that is, we consider dispersion decreasing fibers, where d, g, r, and σ are the parameters of the control system [29,40].

In this special situation, the loss/gain distributed function should be of the form

$$\Gamma(z) = \frac{1}{2}\frac{C_0gd}{(C_0d+g)\exp(gz)-C_0d} + \frac{g-\sigma}{2},$$

and the compression/broadening factor Δ is given by

$$\Delta(z) = \frac{g}{g+C_0d-C_0d\exp(-gz)}.$$

In the particular case when the initial chirp parameter is taken to be $C_0=-g/d$, we get $\Gamma(z)=-\sigma/2$ ($\sigma>0$ for the gain; $\sigma<0$ for the loss), and $\Delta(z)=\exp(gz)$. Then, the width of soliton given by (7) can be exponentially compressed for $g<0$ and exponentially broadened for $g>0$. The maximum amplitude increases for $\sigma>-g$ and decreases for $\sigma<-g$, and it remains unchanged when $\sigma=-g$, as discussed in detail in Ref. [29].

On the other hand, when the soliton amplitude A_s vanishes, the solution $E(z,t)$ given by Eq. (6) reduces to the form $E_c(z,t)=A\Delta(z)\sqrt{D(z)/R(z)}\exp(i\varphi_c)$, which is a cw solution for Eq. (1). Therefore, in general, the exact solution $E(z,t)$ given by Eq. (6) describes a soliton solution embedded in a cw background with the group velocity $V_{sc}=\frac{1}{2}[A_sM_R/M_I+(\omega+\omega_s)]\int_0^z D(\zeta)d\zeta$ [24].

In the following, we will discuss the generation, compression, and propagation of pulse trains based on the general form of the solution (6). We consider the case when $\omega=\omega_s$ and when the cw amplitude A exceeds a critical value $A^2>A_s^2/4$, in other words, when the power of cw light exceeds a quarter of the peak power of the soliton. Then the function $Q(z,t)$ in the solution (6) becomes as follows:

$$Q(z,t) = A - A_s\frac{A_s\cosh\theta - 2A\cos\varphi + iM_R\sinh\theta}{2A\cosh\theta - A_s\cos\varphi}, \tag{9}$$

where

$$\theta = -\frac{1}{2}A_sM_R\Delta(z)\int_0^z D(\zeta)d\zeta - \theta_0,$$

$$\varphi = M_R\Delta(z)t - \omega M_R\Delta(z)\int_0^z D(\zeta)d\zeta - \varphi_0,$$

and $M_R=\sqrt{4A^2-A_s^2}$.

A straightforward analysis reveals that the solution (9) is periodic with the period $L=2\pi/[M_R\Delta(z)]$ in the temporal coordinate and aperiodic in the space variable. Note that the period L is not constant due to the presence of the compression/broadening factor $\Delta(z)$, and it increases or decreases along the propagation direction z depending on the actual z dependence of the compression/broadening factor $\Delta(z)$. Therefore, the exact solution (9) may be considered as describing the *modulation instability* (MI) process in inhomogeneous fibers [44]. In brief, MI is a nonlinear wave phenomenon in which an exponential growth of small perturbations results from the interplay between nonlinearity and GVD. MI has been previously studied in various physical settings, for example, in the context of fluid mechanics (the so-called Benjamin-Feir instability) [45], in nonlinear optics [46], plasma physics [47], and very recently in the field of matter-waves solitons, which is the formation of bright soliton trains in a Bose-Einstein condensate [48].

In order to understand this MI process, we first introduce $\epsilon=\exp(\theta_0)$, which is a small quantity for $\theta_0<0$, we linearize Eq. (6) with respect to ϵ and we get a good approximation (in first order in the small quantity ϵ) for the initial value (at $z=0$) of the solutions (9) and (6) in the form

$$E(0,t) \approx \sqrt{\frac{D(0)}{R(0)}}[\rho + \epsilon\chi\cos(M_Rt-\varphi_0)] \times\exp[i(C_0t^2/2+\omega t)], \tag{10}$$

where $\rho=(2A^2-A_s^2-iA_sM_R)/(2A)$ with $|\rho|=|A|$, and $\chi=A_sM_R(M_R-iA_s)/(2A^2)$.

The numerical results (see below) show that the solution of the initial value problem associated with Eq. (1) with initial condition (10) can be well described by the exact solution (6); that is, a small periodic perturbation of the cw solution may lead to the onset of instability. Based on this approximation, we can conveniently investigate the generation, compression, and propagation of pulse trains produced by the MI process. Indeed, this phenomenon can be used to produce a train of optical solitons of the form (9). However,

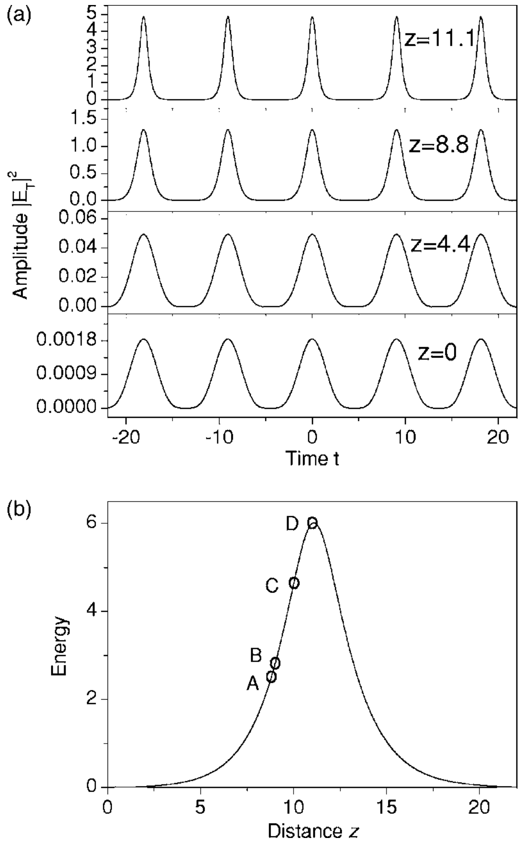

FIG. 1. (a) The intensity distribution plots of the pulse train (11) at different distances $z=0$, $z=4.4$, $z=8.8$, and $z=11.1$; (b) the z dependence of energy of each pulse, for the control system given by Eq. (8). Here the parameters are as follows: $A_s=1.1$, $A=0.65$, $\omega=0$, $d=r=1$, $g=0.01$, $\sigma=0.01$, $\theta_0=-4$, $\varphi_0=0$, and $C_0=0$ (nonchirped input).

this soliton train cannot directly propagate along the fiber because of the presence of the background wave,

$$Q_B(z) = A - A_s \frac{A_s \cosh \theta + 2A + iM_R \sinh \theta}{2A \cosh \theta + A_s}.$$

Subtracting this background field from formula (9), one obtains a new train of pulses with zero background as follows:

$$E_T(z,t) = \Delta(z)\sqrt{\frac{D(z)}{R(z)}} Q_T(z,t)\exp(i\varphi_c), \tag{11}$$

where

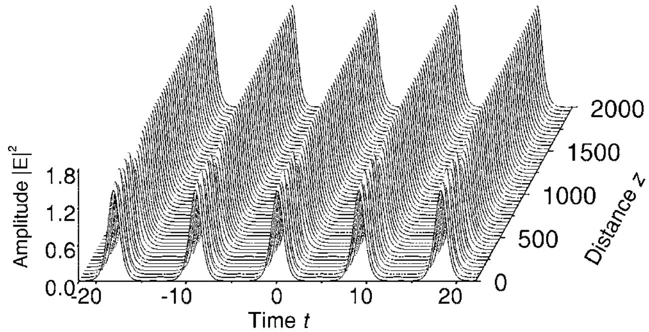

FIG. 2. The evolution plot of the input pulse train (11) corresponding to $z=8.8$ (see Figs. 1(a) and 1(b), and the point A marked in Fig. 1(b)) for a control system given by Eq. (8). Here the parameters are the same as in Fig. 1.

$$Q_T(z,t) = \frac{A_s M_R (M_R \cosh \theta - iA_s \sinh \theta)(1+\cos \varphi)}{(2A \cosh \theta + A_s)(2A \cosh \theta - A_s \cos \varphi)}.$$

Then from Eq. (11), we got that the energy of each pulse in the pulse train is given by

$$\int_0^L |E_T(z,t)|^2\, dt = 2\pi M_R \frac{\Delta(z)D(z)}{R(z)} I(z), \tag{12}$$

where

$$I(z) = \frac{2A \cosh \theta - A_s}{2A \cosh \theta + A_s} - \frac{2A \cosh \theta - 2A_s}{\sqrt{4A^2 \cosh^2 \theta - A_s^2}}.$$

Figure 1(a) presents the intensity distribution plots of a train of five pulses generated by the MI process at different distances z for the exponentially distributed control system given by Eq. (8). Note that for our choice of the parameters ($g=\sigma$), the two exponentially distributed control functions, the GVD distribution $D(z)$, and the nonlinearity distribution $R(z)$ are not dependent. Therefore, the corresponding Wronskian function $W(D,R)$ is zero, resulting in a simpler formula for the corresponding gain/loss function $\Gamma(z)$ [see Eq. (4)]. In Fig. 1(b) we plot the z dependence of the energy (12) of each pulse in the pulse train. We mark by circles in Fig. 1(b) the different energies of the solitons corresponding to $z=8.8$ (point A), $z=9$ (point B), $z=10$ (point C), and $z=11$ (point D), respectively. In the next plots we will consider the propagation of an input train of solitons of different amplitudes (and energies) corresponding to these marked

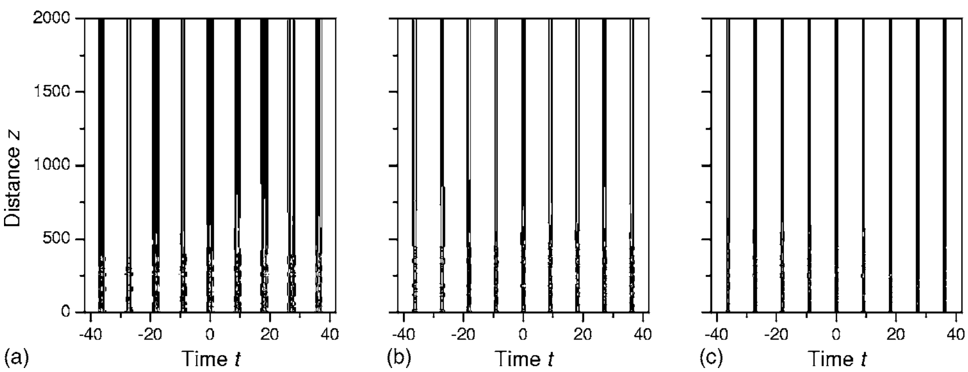

FIG. 3. The contour plot showing the evolution of three different input pulse trains of the form (11) corresponding to (a) $z=9$; (b) $z=10$; (c) $z=11$ (see the points B, C, and D marked in Fig. 1(b)), for the control system given by Eq. (8). Here the parameters are the same as in Fig. 1.

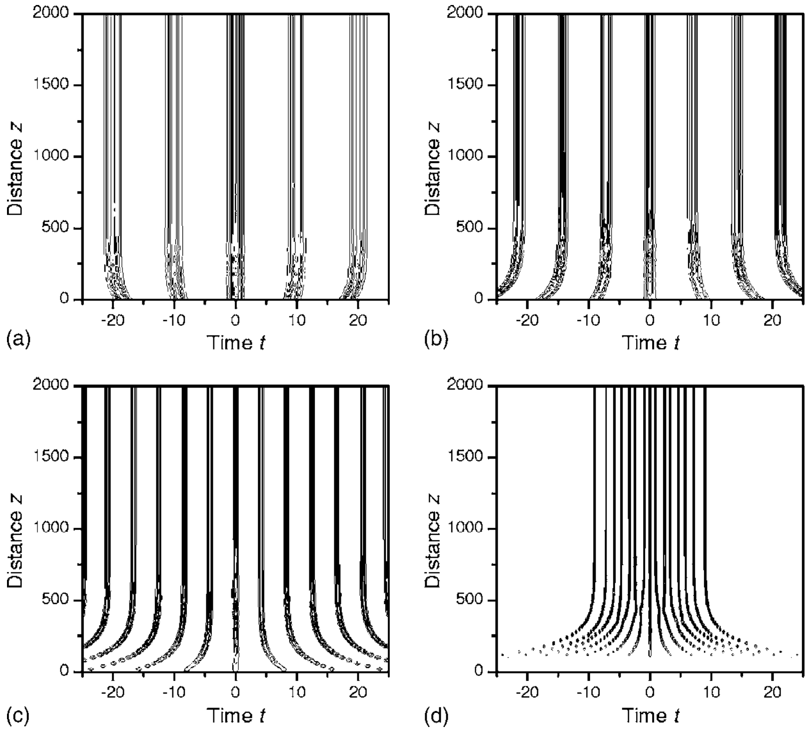

FIG. 4. The contour plot showing the evolution of the pulse train (11) at $z=8.8$ (see the point A marked in Fig. 1(b)) for nonzero input chirp parameter: (a) $C_0=0.001$; (b) $C_0=-0.002$; (c) $C_0=-0.005$; and (d) $C_0=-0.008$ for the same control system given by Eq. (8). Here the parameters are the same as in Fig. 1.

points in Fig. 1(b). From Figs. 1(a) and 1(b), one can see that each pulse in the pulse train (11) is narrowing in width during the first stage of propagation and its energy (its peak value; see Fig. 1(b)) increases until the propagation distance reaches the value $z=z_1$ given by the following implicit equation:

$$\Delta(z_1)\int_0^{z_1} D(\zeta)d\zeta = -\frac{2\theta_0}{A_s M_R}.$$

Note that for our parameter choice this normalized propagation distance is $z_1 \approx 11.1$. Next we will show the output of extensive numerical simulations of the propagation of pulse trains (11) in the inhomogeneous optical fiber. The numerical simulations reveal that the propagation of the pulse train (11) crucially depends on the separation L between pulses and the energy of each pulse.

Figure 2 presents the evolution over 2000 normalized propagation units of the input pulse train (11) composed of five solitons of moderate peak amplitude (see the second row from top in Fig. 1(a) corresponding to $z=8.8$) for the control system given by Eq. (8). From Fig. 2 it can be seen that the pulse train (11) can stably propagate along the fiber, except for some oscillations present at the initial stage of propagation. The separation L between the pulses in the pulse train keeps almost constant: $L=9.069$. In fact, our numerical simulations show that when the energy of each pulse in the pulse train (11) is larger than some critical value, the pulse train (11) can stably propagate along the fiber, as shown in Fig. 3. Thus, when the energy of the pulse is close to its maximum value (see the point D marked in Fig. 1(b)) the pulse train propagates almost undistorted (see panel (c) in Fig. 3 corresponding to the propagation over 2000 normalized units of a train of nine solitons of relatively high peak amplitudes). Also, it is worth noting that in Fig. 3, the initial chirp parameter $C_0=0$ (that is, the pulse is not chirped at input). Next, we consider the effect of the initial nonzero chirp parameter C_0 on soliton train propagation, as shown in Fig. 4. Here we take at input a pulse train consisting of several solitons [corresponding to Eq. (11) for $z=8.8$], and we consider the same control system given by Eq. (8). From Fig. 4, it can be seen that when the initial chirp parameter $C_0>0$, the separation distance between the solitons in the pulse train increases a bit at the beginning of the propagation process, and then keeps unchanged, as shown in Fig. 4(a). When the initial chirp parameter $C_0<0$, the separation distance between the individual solitons is quickly narrowing with the increase of $|C_0|$, eventually leading to focusing of the pulse train, as shown in Figs. 4(b)–4(d). These results show that the propagation of pulse train is quite sensitive to the actual value of the initial chirp parameter.

Furthermore, for the sake of comparison, we also investigated the ideal case when $D(z)=R(z)=1$ and $C_0=0$, which corresponds to the standard NLS equation. At input we con-

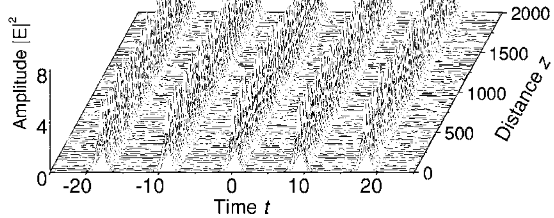

FIG. 5. The intensity evolution plot of the pulse train (11) corresponding to $z=9.5$ for the ideal fiber with $D(z)=R(z)=1$. Here the parameters are as follows: $A_s=1.1$, $A=0.65$, $\omega=0$, $\theta_0=-4$, $\varphi_0=0$, and $C_0=0$ (a nonchirped input).

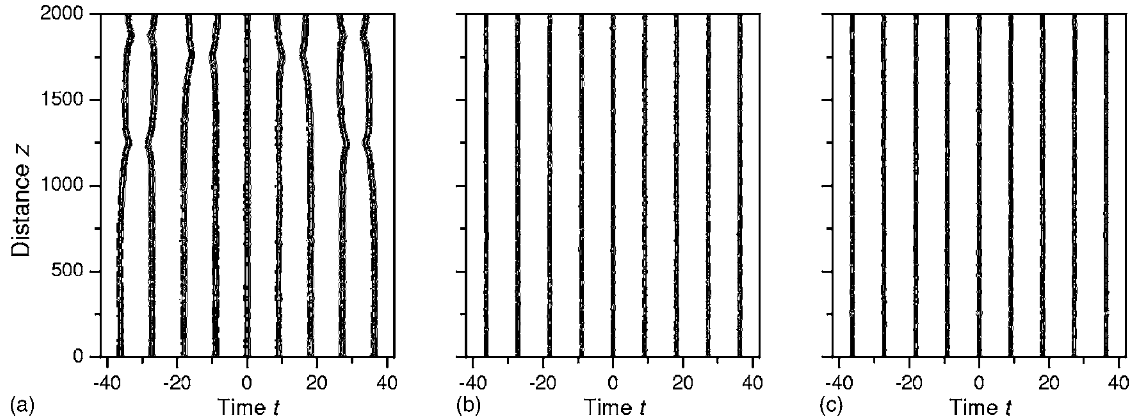

FIG. 6. The contour plots showing the evolution of three different input pulse trains of the form (11) corresponding to (a) $z=9$; (b) $z=10$; (c) $z=11$ for the ideal system with $D(z)=R(z)=1$. Here the parameters are the same as in Fig. 5.

sider the pulse train given by Eq. (11) for $z=9.5$. From Fig. 5 one can see that the pulse train stably propagates along the fiber, except for some persistent oscillations of the soliton's amplitude; however, the separation between solitons in the pulse train keeps almost constant. Also, comparing Fig. 5 with Fig. 2, we find that the combined effects of controlling both the group velocity dispersion distribution and the nonlinearity distribution can restrict to some extent the interaction between the solitons in the pulse train (see Fig. 2). For the sake of completeness, we also present in Fig. 6 the contour plots, showing the evolution over 2000 normalized units in the same ideal conditions as in Fig. 5, of three different input pulse trains. Each input pulse train is composed of nine solitons of different amplitudes and peak energies. Thus comparing, for example, the panel (a) in Fig. 6 [corresponding to the case of the ideal system with $D(z)=R(z)=1$] with panel (a) in Fig. 3 (corresponding to the exponentially distributed control system), we see that the interaction between solitons in the pulse train is greatly reduced when we use a suitable control system, and this may lead to an increase of the transmission bit rate in the optical soliton communications links.

In conclusion, we have considered the nonlinear Schrödinger equation with varying coefficients accounting for distributed group-velocity dispersion, nonlinearity, and gain or loss. By a direct transformation of variables and functions, the exact bright soliton solutions on the continuous-wave background have been found, as expressed in quadratures for a general control fiber system. The extensive numerical simulations have shown that solitons can be compressed at demand by choosing the appropriate control fiber system, and that the pulse train generated by the modulation instability process can propagate stably along the inhomogeneous fiber when the energy of each pulse in the pulse train is larger than some threshold value.

Note added: After the submission of this work a paper by Kruglov *et al.* has been published [49], which reported a broad class of exact self-similar solutions to the nonlinear Schrödinger equation with distributed coefficients describing both bright and dark solitary waves and oscillatory (periodic) solutions expressed in terms of Jacobi elliptic functions, by using a transform of functions and variables similar to that given by Eqs. (2)–(4).

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